1 SAMPLE PAPER [SOLVED]

Time allowed: 3 hours | Maximum marks: 80

General Instructions: *Same as CBSE Sample Question Paper-2024 (Solved).*

SECTION–A

Multiple Choice Questions

Each question carries 1 mark.

1. If $A = [a_{ij}]$ is a skew-symmetric matrix of order n, then

(a) $a_{ij} = \frac{1}{a_{ji}}$ for $i \neq j$ (b) $a_{ij} = a_{ji}$ for $i \neq j$

(c) $a_{ij} = 0$, for $i = j$ (d) none of these

2. If A is a non-singular matrix of order 3 then $|AA^{-1}|$ is equal to

(a) 1 (b) $|A|$ (c) $|A^{-1}|$ (d) –1

3. Area of a parallelogram with vertices A, B, C and D is given by

(a) $|\overrightarrow{AB} \times \overrightarrow{AD}|$ (b) $|\overrightarrow{AB} \times \overrightarrow{CD}|$ (c) $|\overrightarrow{AD} \times \overrightarrow{BC}|$ (d) None of these

4. The function $f(x) = [x]$, where $[x]$ denotes the greatest integer function, is continuous at

(a) 4 (b) –2 (c) 1 (d) 1.5

5. The anti derivative of $\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)$ equals

(a) $\frac{1}{3}x^{1/3} + 2x^{1/2} + C$ (b) $\frac{2}{3}x^{2/3} + \frac{1}{3}x^2 + C$

(c) $\frac{2}{3}x^{3/2} + 2x^{1/2} + C$ (d) $\frac{3}{2}x^{3/2} + \frac{1}{2}x^{1/2} + C$

6. The degree of the differential equation $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}} = \frac{d^2y}{dx^2}$ is

(a) 4 (b) $\frac{3}{2}$ (c) not defined (d) 2

7. **A point out of following points lie in plane represented by $2x + 3y \leqslant 12$ is**

(a) (0, 3) (b) (3, 3) (c) (4, 3) (d) (0, 5)

8. **The unit vector in the direction of the sum of the vectors $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} + \hat{j} + 3\hat{k}$ is**

(a) $\frac{1}{\sqrt{26}}\hat{i} + \frac{5}{\sqrt{26}}\hat{k}$ (b) $\frac{1}{\sqrt{3}}\hat{i} - \frac{5}{\sqrt{26}}\hat{k}$ (c) $\frac{1}{\sqrt{10}}\hat{i} + \frac{1}{\sqrt{20}}\hat{k}$ (d) $\frac{1}{\sqrt{32}}\hat{i} - \frac{5}{\sqrt{3}}\hat{k}$

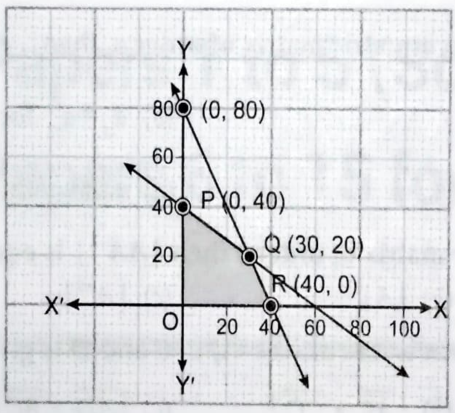
9. $\int_1^{\sqrt{3}} \frac{dx}{1+x^2}$ **is equal to**

(a) $\frac{\pi}{3}$ (b) $\frac{2\pi}{3}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{12}$

10. **The inverse of the matrix** $\begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$ **is**

(a) $\begin{bmatrix} 4 & 1 \\ -3 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 4/11 & 1/11 \\ -3/11 & 2/11 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$

11. **For an *LPP* the objective function is $Z = 4x + 3y$ and the feasible region determined by a set of constraints (linear inequations) is shown in the graph.**



Which of the following statements is true?

(a) Maximum value of Z is at R.

(b) Maximum value of Z is at Q.

(c) Value of Z at R is less than the value at P.

(d) The value of Z at Q is less than the value at R.

12. **If** $\begin{vmatrix} 4 & 0 \\ 0 & 1 \end{vmatrix} = \begin{vmatrix} x & 0 \\ 1 & 2x \end{vmatrix}$**, then the possible values of 'x' is/are**

(a) 2 (b) $\sqrt{2}$ (c) $-\sqrt{2}$ (d) $\sqrt{2}, -\sqrt{2}$

13. **If *A* is a square matrix of order (3 × 3) such that $|A| = 2$. Then *adj* (*adj A*) is**

(a) 2A (b) A (c) –A (d) None of these

14. **If $P(B) = \frac{3}{5}$, $P(A/B) = \frac{1}{2}$ and $P(A \cup B) = \frac{4}{5}$, then $P((A \cup B)') + P(A' \cup B)$ is equal to**

(a) $\frac{1}{5}$ (b) $\frac{9}{5}$ (c) $\frac{7}{2}$ (d) 1

15. The solution of differential equation $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$ is

(a) $y = \tan^{-1} x$ (b) $y - x = k(1 + xy)$ (c) $x = \tan^{-1} y$ (d) $\tan(xy) = k$

16. If $f(x) = e^{x^2}$ then $f'(x)$ is equal to

(a) $2e^{x^2}(1 + 2x^2)$ (b) $e^{x^2}(1 + 2x^2)$ (c) $2e^{x^2}(1 + 2x)$ (d) $e^{x^2}(1 + 2x)$

17. If $\vec{a}$ and $\vec{b}$ are unit vectors, then the angle between $\vec{a}$ and $\vec{b}$ for $\sqrt{3}\,\vec{a} - \vec{b}$ to be a unit vector is

(a) 30° (b) 45° (c) 60° (d) 90°

18. P is the point on the line segment joining the points (3, 2, –1) and (6, 2, –2). If x co-ordinate of P is 5, then its y co-ordinate is

(a) 2 (b) 1 (c) – 1 (d) – 2

Assertion-Reason Based Questions

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) *Both A and R are true and R is the correct explanation of A.*

(b) *Both A and R are true but R is not the correct explanation of A.*

(c) *A is true but R is false.*

(d) *A is false but R is true.*

19. **Assertion (A) :** The domain of the function $\text{cosec}^{-1}(2x)$ is $\left(-\infty, -\frac{1}{2}\right] \cup \left[\frac{1}{2}, \infty\right)$

Reason (R) : $\text{cosec}^{-1}(-2) = -\frac{\pi}{6}$

20. **Assertion (A) :** Direction cosines of vector $\vec{a} = \hat{i} + \hat{j} - 2\hat{k}$ are $\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}$.

Reason (R) : If vector $\vec{r} = a\hat{i} + b\hat{j} + c\hat{k}$ then its direction ratios are $\frac{a}{|\vec{r}|}, \frac{b}{|\vec{r}|}$ and $\frac{c}{|\vec{r}|}$, where $|\vec{r}| = \sqrt{a^2 + b^2 + c^2}$.

SECTION–B

(This section comprises of very short answer type questions (VSA) of 2 marks each.)

21. Find the value of $\sin^{-1}\left(\cos\left(\frac{43\pi}{5}\right)\right)$.

OR

Let $f: \mathbb{R} - \left\{-\frac{4}{3}\right\} \to \mathbb{R}$ be a function defined as $f(x) = \frac{4x}{3x+4}$. Show that,

$f: \mathbb{R} - \left\{-\frac{4}{3}\right\} \to$ Range of f, is one-one and onto.

22. A stone is dropped into a quiet lake and waves move in circles at the speed of 5 cm/s. At the instant when the radius of the circular wave is 8 cm, how fast is the enclosed area increasing?

23. Let $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$ be two vectors. Show that the vectors $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$ are perpendicular to each other.

OR

Find the direction cosines of a line passing through the point (1, 3, 5) and (2, 4, 6).

24. If $y = \text{cosec}(\cot\sqrt{x})$ then find $\frac{dy}{dx}$.

25. The vectors $\vec{a} = 3\hat{i} + x\hat{j}$ and $\vec{b} = 2\hat{i} + \hat{j} + y\hat{k}$ are mutually perpendicular. If $|\vec{a}| = |\vec{b}|$, then find the value of y.

SECTION–C

(This section comprises of short answer type questions (SA) of 3 marks each.)

26. Find: $\int \frac{dx}{\sqrt{5 - 4x - 2x^2}}$

27. Assume that each born child is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls given that

(*i*) the youngest is a girl? (*ii*) atleast one is a girl?

OR

In a hockey match, both teams *A* and *B* scored same number of goals up to the end of the game, so to decide the winner, the referee asked both the captains to throw a die alternately and decided that the team, whose captain gets a six first, will be declared the winner. If the captain of team *A* was asked to start, find their respective probabilities of winning the match and state whether the decision of the referee was fair or not.

28. Evaluate: $\int_0^{\pi} \frac{4x \sin x}{1 + \cos^2 x} dx$

OR

Evaluate: $\int_{-1}^{2} |x^3 - x|\, dx$

29. Find the general solution of the following differential equation:

$$xdy - (y + 2x^2)dx = 0$$

OR

Find the general solution of the following differential equation:

$$x\cos\left(\frac{y}{x}\right)\frac{dy}{dx} = y\cos\left(\frac{y}{x}\right) + x$$

30. Solve the following linear programming problem (LPP) graphically.

Maximize $Z = x + 2y$

subject to constraints;

$x + 2y \geq 100$

$2x - y \leq 0$

$2x + y \leq 200$

$x, y \geq 0$

31. Find: $\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}\,dx$

SECTION–D

(This section comprises of long answer type questions (LA) of 5 marks each.)

32. Find the area of the triangle whose vertices are (–1, 1), (0, 5) and (3, 2), using integration.

33. Check whether the relation R in the set Z of integers defined as $R = \{(a, b) : a + b$ is "divisible by 2"$\}$ is reflexive, symmetric or transitive. Write the equivalence class containing 0, *i.e.*, [0].

OR

Check whether the relation R in $\mathbb{R}$ defined by $R = \{(a, b) : a \le b^3\}$ is reflexive, symmetric or transitive.

34. Find the shortest distance between the lines

$$\vec{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k}) \text{ and } \vec{r} = 5\hat{i} - 2\hat{j} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$$

If the lines intersect find their point of intersection.

OR

Find the vector and cartesian equations of the line passing through the point (1, 2, –4) and perpendicular to the two lines $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$.

35. If $A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$, find A^{-1}.

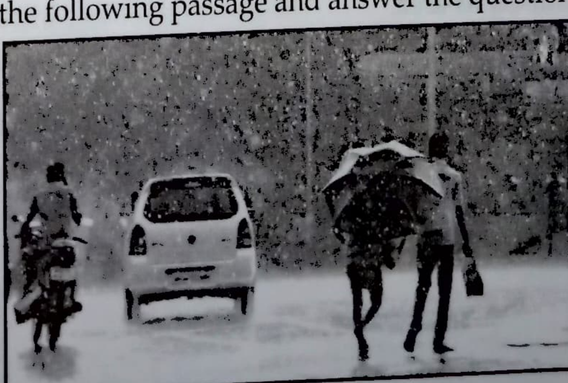
Hence solve the system of equations:

$$x - 2y = 10$$
$$2x - y - z = 8$$
$$-2y + z = 7$$

SECTION–E

(This section comprises of 3 case-study/passage-based questions of 4 marks each. First two case study questions have three sub-parts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two sub-parts of 2 marks each.)

36. **Case-Study 1:** Read the following passage and answer the questions given below.



The temperature of some days during rainy season is given by $f(x) = -0.1x^2 + mx + 34.5, 0 \leq x \leq 15$, m be a constant, where $f(x)$ is the temperature in °C at x-days.

(*i*) Is the function differentiable in the interval (0, 15)? Justify your answer.

(*ii*) If 2 is the critical point of the function, then find the value of the constant m.

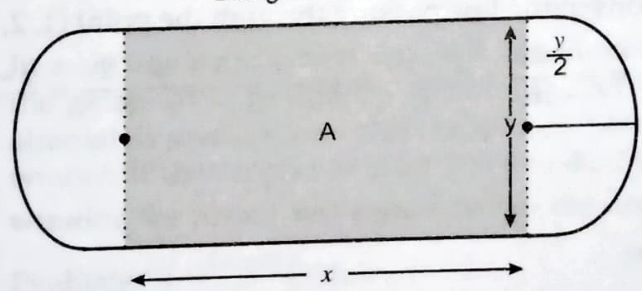
(*iii*) Find the intervals in which the function is strictly increasing/strictly decreasing.

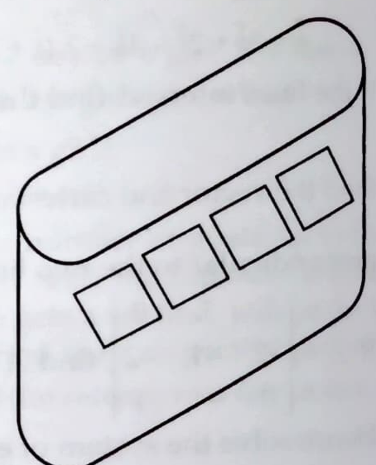
OR

Find the points of local maxima/local minima, if any, in the interval (0, 15) as well as points of absolute maxima/absolute minima in the interval [0, 15]. Also find the corresponding local maximum/local minimum and absolute maximum/absolute minimum values of the function.

37. **Case-Study 2:** Read the following passage and answer the questions given below.

Design of Floor





Building

An architect designs a building for a multi-national company. The floor consists of a rectangular region with semicircular ends having a perimeter of 200 m as shown above in figure:

(*i*) If x and y represents the length and breadth of the rectangular region, then find the area function (A) in terms of x.

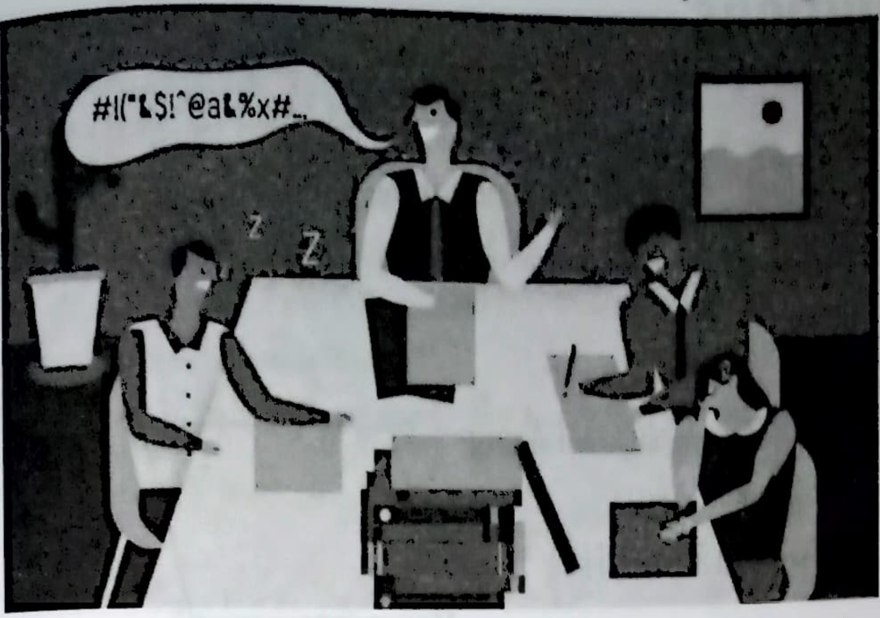
(*ii*) Find the critical point of the function (A).

(*iii*) Use First Derivative Test to find the length x and breadth y of the rectangular region that maximised its area.

OR

Use Second Derivative Test to find the length x and breadth y of the rectangular region that maximised its area. Also find the maximum area.

38. **Case-Study 3:** Read the following passage and answer the questions given below.



In an office three employees Vinay, Sonia and Iqbal process incoming copies of a certain form. Vinay processes 50% of the forms. Sonia processes 20% and Iqbal the remaining 30% of the forms. Vinay has an error rate of 0.06, Sonia has an error rate of 0.04 and Iqbal has an error rate of 0.03.

(*i*) Find the total probability of committing an error in processing the form.

(*ii*) The manager of the company wants to do a quality check. During inspection he selects a form at random from the days output of processed forms. If the form selected at random has an error, find the probability that the form is NOT processed by Vinay.