3 SAMPLE PAPER [SOLVED]

Time allowed: 3 hours **Maximum marks: 80**

General Instructions: *Same as CBSE Sample Question Paper-***2024** *(Solved).*

SECTION–A

Multiple Choice Questions

Each question carries 1 *mark.*

1. If $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$, then $A^2 - 5A$ is

(a) I (b) $14\,I$

(c) 0 (d) None of these

2. The value of $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 3A - B$ then the values of A and B are

(a) $A = 2abc, B = a + b + c$ (b) $A = 0, B = a^2 + b^2 + c^2$

(c) $A = 3abc, B = a + b + c$ (d) $A = abc, B = a^3 + b^3 + c^3$

3. The two vectors $\hat{j} + \hat{k}$ and $3\hat{i} - \hat{j} + 4\hat{k}$ represents the two sides AB and AC, respectively of ΔABC. The length of the median through A is

(a) $\dfrac{\sqrt{34}}{2}$ (b) $\dfrac{\sqrt{48}}{2}$

(c) $\sqrt{18}$ (d) None of these

4. If the function $f(x)$ defined by

$$f(x) = \begin{cases} \dfrac{\log(1 + ax) - \log(1 - bx)}{x}, & \text{if } x \neq 0 \\ k, & \text{if } x = 0 \end{cases}$$

is continuous at $x = 0$, then the value of k is

(a) a (b) b

(c) $a + b$ (d) 0

5. If $f'(x) = 4x^3 - \frac{3}{x^4}$ such that $f(2) = 0$. Then $f(x)$ is

(a) $x^4 + \frac{1}{x^3} - \frac{129}{8}$

(b) $x^3 + \frac{1}{x^4} + \frac{129}{8}$

(c) $x^4 + \frac{1}{x^3} + \frac{129}{8}$

(d) $x^3 + \frac{1}{x^4} - \frac{129}{8}$

6. The order and degree of the differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{\frac{1}{4}} + x^{1/5} = 0$ respectively, are

(a) 2 and 4

(b) 2 and 2

(c) 2 and 3

(d) 3 and 3

7. Consider a LPP given by

Min $Z = 6x + 10y$

subject to $x \geq 6; y \geq 2; 2x + y \geq 10; x, y \geq 0$

Redundant constraints in this LPP are

(a) $x \geq 0, y \geq 0$

(b) $x \geq 6, 2x + y \geq 10$

(c) $2x + y \geq 10$

(d) none of these

8. Let the vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}| = 3$ and $|\vec{b}| = \frac{\sqrt{2}}{3}$, then $\vec{a} \times \vec{b}$ is a unit vector, if the angle between $\vec{a}$ and $\vec{b}$ is

(a) $\pi / 6$

(b) $\pi / 4$

(c) $\pi / 3$

(d) $\pi / 2$

9. The value of $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \operatorname{cosec} x \, dx$ is

(a) $\log\left(\frac{\sqrt{2} - 1}{2 - \sqrt{3}}\right)$

(b) $\log \sqrt{2}$

(c) $\log\left(\frac{\sqrt{2} + 1}{2 + \sqrt{3}}\right)$

(d) $\log\left(\frac{\sqrt{2} - 1}{2 + \sqrt{3}}\right)$

10. If $A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$ then the value of $A^T A^{-1}$ is

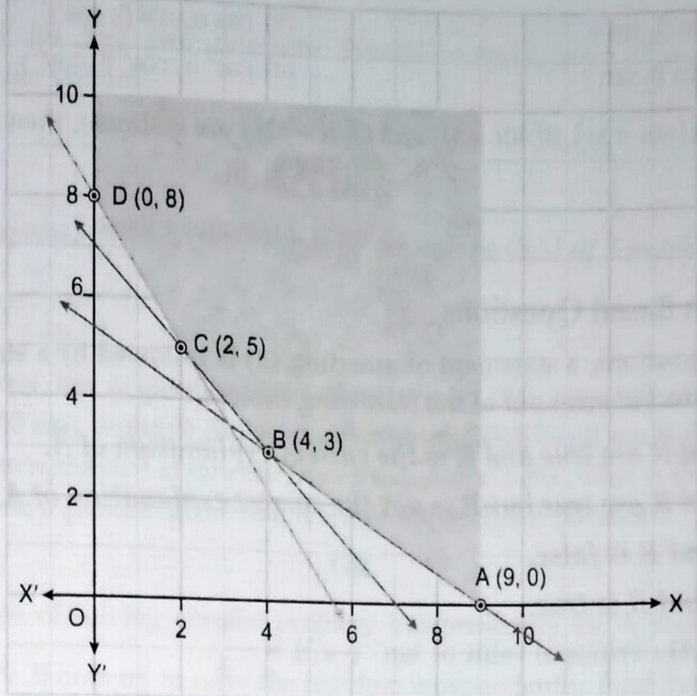
(a) $\begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}$

(b) $\begin{bmatrix} \cos x & \sin x \\ 1 & 0 \end{bmatrix}$

(c) A'

(d) Zero matrix

11. **Feasible region (shaded) for a LPP is shown in the given figure. Minimum of $Z = 4x + 3y$ occurs at the point.**



(*a*) (0, 8) (*b*) (2, 5) (*c*) (4, 3) (*d*) (9, 0)

12. **If C_{ij} denotes the cofactor of element P_{ij} of the matrix $P = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & 2 & 4 \end{bmatrix}$, then the value of $C_{31} \cdot C_{23}$ is**

(*a*) 5 (*b*) 24 (*c*) –24 (*d*) –5

13. **If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ then A^{-1} is**

(*a*) $\begin{bmatrix} 2 & -3 \\ 5 & 2 \end{bmatrix}$ (*b*) $\begin{bmatrix} -2 & 4 \\ 3 & 5 \end{bmatrix}$ (*c*) $\frac{1}{19}\begin{bmatrix} 3 & -2 \\ 5 & 3 \end{bmatrix}$ (*d*) $\frac{1}{19}A$

14. **If A and B are two events such that $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$ and $P(A/B) = \frac{1}{4}$, then $P(A' \cap B')$ is equals to**

(*a*) $\frac{1}{12}$ (*b*) $\frac{3}{4}$ (*c*) $\frac{1}{4}$ (*d*) $\frac{3}{16}$

15. **The solution of $\frac{dy}{dx} + y = e^{-x}$; $y(0) = 0$ is**

(*a*) $y = e^{x}(x - 1)$ (*b*) $y = x e^{-x}$ (*c*) $y = x e^{-x} + 1$ (*d*) $y = (x + 1)e^{-x}$

16. **If $y = \tan^{-1} x$ then the value of $(1 + x^2)y_2$ is equal to**

(*a*) xy_1 (*b*) $-xy_1$ (*c*) $2xy_1$ (*d*) $-2xy_1$

17. If α, β, γ, are the angles that a line makes with a positive direction of x, y, z axes, respectively, then the direction cosines of the line are

(*a*) $\sin\alpha, \sin\beta, \sin\gamma$ (*b*) $\cos\alpha, \cos\beta, \cos\gamma$

(*c*) $\tan\alpha, \tan\beta, \tan\gamma$ (*d*) $\cos^2\alpha, \cos^2\beta, \cos^2\gamma$

18. If points $A(60\hat{i} + 3\hat{j})$, $B(40\hat{i} - 8\hat{j})$ and $C(a\hat{i} - 52\hat{j})$ are collinear, then a is equal to

(*a*) 40 (*b*) 50

(*c*) -40 (*d*) none of these

Assertion-Reason Based Questions

In the following questions, a statement of assertion (*A*) is followed by a statement of Reason (*R*). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

19. Assertion (*A*) : Principal value of $\tan^{-1}(-\sqrt{3})$ is $-\frac{\pi}{3}$.

Reason (*R*) : $\tan^{-1}x : \mathbb{R} \to \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ so for any $x \in \mathbb{R}$, $\tan^{-1}(x)$ represents an angle in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

20. Assertion (*A*) : Direction cosines of z-axis are 0, 0, 1.

Reason (*R*) : If l, m, n be the direction cosines of a line then $l^2 + m^2 + n^2 = 1$.

SECTION–B

(This section comprises of very short answer type questions (VSA) of 2 marks each.)

21. Find the value of $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$.

OR

Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ such that $f(x) = x^3$.

Check whether f is bijective or not.

22. A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2 cm/s. How fast is its height on the wall decreasing when the foot of the ladder is 4 m away from the wall?

23. Find a unit vector perpendicular to both the vectors $\vec{a}$ *and* $\vec{b}$, where $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$.

OR

If the x-coordinate of a point P on the joining of $Q(2, 2, 1)$ and $R(5, 1, -2)$ is 4, then find its z-coordinate.

24. If $x^{13}y^7 = (x + y)^{20}$, then prove that $\frac{dy}{dx} = \frac{y}{x}$.

25. Show that for any two non-zero vectors $\vec{a}$ and $\vec{b}$, $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$ if $\vec{a}$ and $\vec{b}$ are perpendicular vectors.

SECTION-C

(This section comprises of short answer type questions (SA) of 3 marks each.)

26. Find: $\int \frac{1}{\sqrt{x^2 + 2x + 2}} dx$

27. An instructor has a question bank consisting of 300 easy true/false questions, 200 difficult, 500 easy multiple choice questions and 400 difficult multiple choice questions. If a question is selected at random from the question bank, what is the probability that it will be an easy question given that it is a multiple choice question?

OR

Probabilities of solving specific problem independently by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. If both try to solve the problem independently. Find the probability that (*i*) the problem is solved (*ii*) exactly one of them solved the problem.

28. Evaluate: $\int_{-5}^{5} | x + 2 | \, dx$

OR

Evaluate: $\int_0^{\pi/4} \log(1 + \tan x) dx$

29. Find the particular solution of the differential equation

$(1 + e^{2x})dy + (1 + y^2)\, e^x\, dx = 0$ given that $y = 1$ when $x = 0$.

OR

Show that the differential equation is homogeneous and solve it.

$$\left(1 + e^{\frac{x}{y}}\right)dx + e^{\frac{x}{y}}\left(1 - \frac{x}{y}\right)dy = 0$$

30. Maximize: $Z = -x + 2y$

Subject to: $x \geq 3, x + y \geq 5, x + 2y \geq 6, y \geq 0$.

31. Find: $\int \frac{1 - x^2}{x(1 - 2x)} dx$

SECTION-D

(This section comprises of long answer type questions (LA) of 5 marks each.)

32. Find the area of the region in the first quadrant enclosed by x-axis, line $x = \sqrt{3}y$ and the circle $x^2 + y^2 = 4$.

33. Let $\mathbb{N}$ denotes the set of all natural numbers and R be the relation on $\mathbb{N} \times \mathbb{N}$ defined by $(a, b)\ R\ (c, d)$ if $ad(b + c) = bc(a + d)$. Show that R is an equivalence relation.

OR

Determine whether the relation R defined on the set $\mathbb{R}$ of all real numbers as $R = \{(a, b) : a, b \in \mathbb{R}$ and $a - b + \sqrt{3} \in S$, where S is the set of all irrational numbers$\}$, is reflexive, symmetric and transitive.

34. Find the vector and cartesian equations of the line which is perpendicular to the lines with equations $\frac{x+2}{1} = \frac{y-3}{2} = \frac{z+1}{4}$ and $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and passes through the point (1, 1, 1). Also find the angle between the given lines

OR

Find the shortest distance between the lines :

$\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$ and $\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$

35. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix}$. Find A^{-1}. Hence, solve the system of equations.

$x + y + z = 6, x + 2z = 7, 3x + y + z = 12$.

SECTION–E

(This section comprises of 3 case-Study/passage-based questions of 4 marks each. First two case study questions have three sub-parts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two sub-parts of 2 marks each.)

36. **Case Study 1:** Read the following passage and answer the questions given below.

A teacher of class XII gives home work to his students is a function

$f(x) = 3x^4 + 4x^3 - 12x^2 + 12, -3 \leq x \leq 3$.

(*i*) Is this function differentiable in (–3, 3)?

(*ii*) Find the critical points.

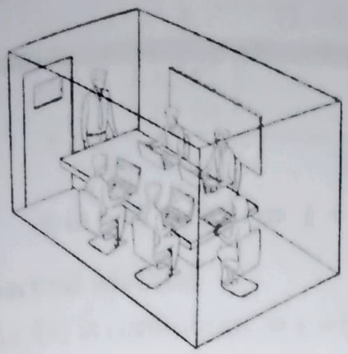
(*iii*) Find the intervals in which the function strictly increasing/decreasing.

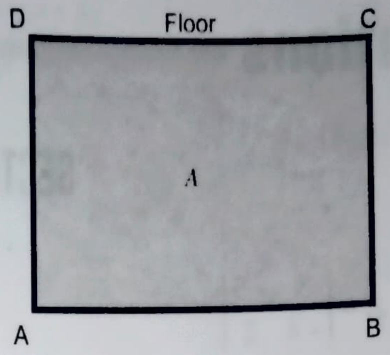
OR

Find the points of local maxima/local minima of the function as well as the points of absolute maxima/absolute minima in the interval [–3, 3]. Also find the corresponding local maximum/local minimum and the absolute maximum/ absolute minimum.

37. **Case Study 2:** Read the following passage and answer the questions given below.

A rectangular hall is to be developed for a meeting of farmers in an agriculture college to aware them for new technique in cultivation. It is given that the floor has a fixed perimeter *P* as shown below.





(*i*) If x and y represents the length and breadth of the rectangular region, then find the area function A in terms of x.

(*ii*) Find the critical point of the function A.

(*iii*) Use First Derivative Test to find the length x and breadth y of rectangular hall that maximize its area.

OR

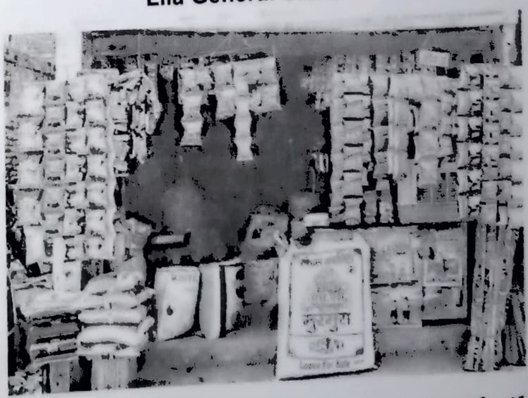
Use Second Derivative Test to find the length x and breadth y of the rectangular hall that maximize the area.

38. **Case Study 3:** Read the following passage and answer the questions given below.

There are two shops in a village market named Lila general store and Mina general store. In Lila general store 30 tin pure Mustard oil, 40 tin adulterated mustard oil while in Mina general store 50 tin pure mustard oil and 60 tin adulterated oil are there. Subhash Babu wants to purchase one tin oil from any shop selecting at random.

Mina General Store

Lila General Store



(*i*) Find the total probability of purchasing adultrated mustard oil if shop and tin of oil are selected at random.

(*ii*) Subhash Babu wants to know quality of mustard oil. Before purchasing he selected first, a shop at random and then selected a tin of mustard oil at random. If the tin selected at random has adulterated oil, find the probability that the selected tin is of Lila general store.