4 SAMPLE PAPER [SOLVED]

Time allowed: 3 hours

General Instructions: *Same as CBSE Sample Question Paper 2024 (Solved).* **Maximum marks: 80**

SECTION-A

Multiple Choice Questions

Each question carries 1 mark.

1. If $A = \begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix}$ and $kA = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$, then the values of k, a, b are

(a) $-6, -12, -18$ (b) $-6, 4, 9$ (c) $-6, -4, -9$ (d) $-6, 12, 18$

2. If A is a square matrix of order 3, $|A| = -5$, then $|A^{-1}|$ is equal to

(a) $\frac{1}{5}$ (b) $-\frac{1}{5}$ (c) 5 (d) -5

3. If $f(x) = \begin{cases} 1, & \text{if } x \leq 3 \\ ax + b, & \text{if } 3 < x < 5 \\ 7, & \text{if } 5 \leq x \end{cases}$ then the values of a and b so that $f(x)$ is continuous are

(a) $a = 3, b = 3$ (b) $a = 3, b = 4$ (c) $a = 3, b = -8$ (d) None of these

4. If $f'(x) = 3x^2 - \frac{4}{x^3}$

(a) $x^3 + \frac{2}{x^2} + C$ (b) $x^3 - \frac{2}{x^2} + C$ (c) $-x^3 + \frac{2}{x^2} + C$ (d) None of these

5. The function $f : R \to R$ given by $f(x) = -|x - 1|$ is

(a) continuous as well as differentiable at $x = 1$

(b) not continuous but differentiable at $x = 1$

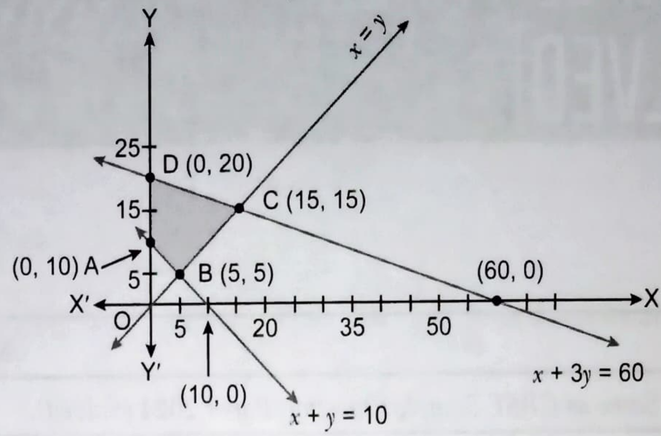
(c) continuous but not differentiable at $x = 1$

(d) neither continuous nor differentiable at $x = 1$

6. Which of the following is not a homogeneous function of x and y?

(a) $x^2 + 2xy$ (b) $2x - y$ (c) $\cos^2\left(\frac{y}{x}\right) + \frac{y}{x}$ (d) $\sin x - \cos y$

7. **Based on the given shaded region as the feasible region in the graph, at which point(s) is the objective function $Z = 3x + 9y$ maximum?**



(*a*) Point B (*b*) Point C

(*c*) Point D (*d*) every point on the line segment CD

8. **If $\vec{a}$ is a nonzero vector of magnitude a and λ a nonzero scalar, then $\lambda\vec{a}$ is unit vector if**

(*a*) $\lambda = 1$ (*b*) $\lambda = -1$ (*c*) $a = |\lambda|$ (*d*) $a = 1/|\lambda|$

9. **The value of $\int_2^8 \frac{\sqrt{10-x}}{\sqrt{x}+\sqrt{10-x}}\,dx$ is equal to**

(*a*) 3 (*b*) –3 (*c*) 5 (*d*) 0

10. **If $A = \begin{bmatrix} 1 & -2 \\ 3 & 0 \end{bmatrix}, B = \begin{bmatrix} -1 & 4 \\ 2 & 3 \end{bmatrix}, C = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ then $5A - 3B - 2C$ is**

(*a*) $\begin{bmatrix} 8 & 20 \\ 7 & 9 \end{bmatrix}$ (*b*) $\begin{bmatrix} 8 & -20 \\ 7 & -9 \end{bmatrix}$ (*c*) $\begin{bmatrix} -8 & 20 \\ -7 & 9 \end{bmatrix}$ (*d*) $\begin{bmatrix} 8 & 7 \\ -20 & -9 \end{bmatrix}$

11. **In a linear programming problem, the constraints on the decision variables x and y are $x - 3y \geq 0, y \geq 0, 0 \leq x \leq 3$. The feasible region**

(*a*) is not in the first quadrant (*b*) is bounded in the first quadrant

(*c*) is unbounded in the first quadrant (*d*) does not exist

12. **Value of k, for which $A = \begin{bmatrix} k & 8 \\ 4 & 2k \end{bmatrix}$ is a singular matrix is**

(*a*) 4 (*b*) – 4 (*c*) ± 4 (*d*) 0

13. **Given that $A = [a_{ij}]$ is a square matrix of order 3 × 3 and $|A| = -7$, then the value of $\sum_{i=1}^{3} a_{i2}A_{i2}$, where A_{ij} denotes the cofactor of element a_{ij}, is**

(*a*) 7 (*b*) – 7 (*c*) 0 (*d*) 49

14. **If A and B are two events such that $P(A) \neq 0$ and $P(B/A) = 1$, then**

(*a*) $A \subset B$ (*b*) $B \subset A$ (*c*) $B = \phi$ (*d*) $A = \phi$

15. **The solution of differential equation $\tan y \sec^2 x\,dx + \tan x \sec^2 y\,dy = 0$ is**

(*a*) $\tan x + \tan y = k$ (*b*) $\tan x - \tan y = k$ (*c*) $\frac{\tan x}{\tan y} = k$ (*d*) $\tan x . \tan y = k$

16. Value of $\dfrac{d^2y}{dx^2}$, if $x = at^2, y = 2at$ is

(a) $\dfrac{-1}{2at^3}$ (b) $\dfrac{1}{2at^2}$ (c) $\dfrac{-1}{2at^2}$ (d) 0

17. In the triangle ABC, which of the following is not true?

(a) $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0}$ (b) $\overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{AC} = \vec{0}$

(c) $\overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{CA} = \vec{0}$ (d) $\overrightarrow{AB} - \overrightarrow{CB} + \overrightarrow{CA} = \vec{0}$

18. The vector equation of the line through the points (3, 4, – 7) and (1, – 1, 6) is

(a) $\vec{r} = 3\hat{i} + 4\hat{j} - 7\hat{k} + \lambda(3\hat{i} + 7\hat{j} + 2\hat{k})$ (b) $\vec{r} = (3\hat{i} + 4\hat{j} - 7\hat{k}) + \lambda(-2\hat{i} - 5\hat{j} + 13\hat{k})$

(c) $\vec{r} = 3\hat{i} + 4\hat{j} - 7\hat{k} + \lambda(3\hat{i} - 7\hat{j} + 5\hat{k})$ (d) $\vec{r} = (3\hat{i} + 4\hat{j} - 7\hat{k}) + \lambda(2\hat{i} + 5\hat{j} - 13\hat{k})$

Assertion-Reason Based Questions

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) *Both A and R are true and R is the correct explanation of A.*

(b) *Both A and R are true but R is not the correct explanation of A.*

(c) *A is true but R is false.*

(d) *A is false but R is true.*

19. **Assertion** (*A*) : $\sin^{-1}(-x) = -\sin^{-1} x;\ x \in [-1, 1]$

Reason (*R*) : $\sin^{-1} : [-1, 1] \to \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$ is a bijection map.

20. **Assertion** (*A*) : Parametric equation of a line $\dfrac{x-1}{2} = \dfrac{y-2}{3} = \dfrac{z}{4}$ are $x = 1 + 2\lambda, y = 2 + 3\lambda, z = 4\lambda$, where λ is a parameter.

Reason (*R*) : The parametric equation of the line $\dfrac{x - x_1}{a} = \dfrac{y - y_1}{b} = \dfrac{z - z_1}{c}$ are $x = x_1 + a\lambda, y = y_1 + b\lambda, z = z_1 + c\lambda$ where λ is a parameter.

SECTION–B

(This section comprises of very short answer type questions (VSA) of 2 marks each.)

21. Write the following function in the simplest form:

$$\tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right), -\frac{\pi}{4} < x < \frac{3\pi}{4}$$

OR

Let $f : \mathrm{N} \to \mathrm{N}$ given by $f(x) = x^2$, check injectivity and surjectivity of f.

22. Sand is pouring from a pipe at the rate of 12 cm^3/s. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is 4 cm?

23. If $\hat{a}$ and $\hat{b}$ are unit vectors, then prove that

$|\hat{a}+\hat{b}| = 2\cos\frac{\theta}{2}$, where θ is the angle between them.

OR

Find the direction cosines of the following line:

$$\frac{3-x}{-1} = \frac{2y-1}{2} = \frac{z}{4}$$

24. Find $\frac{dy}{dx}$ at $x = 1, y = \frac{\pi}{4}$ if $\sin^2 y + \cos xy = K$.

25. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{a}.\vec{b} = 1$ and $\vec{a} \times \vec{b} = \hat{j} - \hat{k}$, then find $|\vec{b}|$.

SECTION–C

(This section comprises of short answer type questions (SA) of 3 marks each.)

26. Find: $\int \frac{x+1}{(x^2+1)x} dx$

27. Assume that the chances of a patient having a heart attack is 40%. Assuming that a meditation and yoga course reduces the risk of heart attack by 30% and prescription of certain drug reduces its chance by 25%. At a time a patient can choose any one of the two options with equal probabilities. It is given that after going through one of the two options, the patient selected at random suffers a heart attack. Find the probability that the patient followed a course of meditation and yoga.

OR

A class has 15 students whose ages are 14, 17, 15, 14, 21, 17, 19, 20, 16, 18, 20, 17, 16, 19 and 20 years. One student is selected in such a manner that each has the same chance of being chosen and the age X of the selected student is recorded. What is the probability distribution of the random variable X? Find mean of X?

28. Evaluate: $\int_0^{\pi/4} \frac{\sin x + \cos x}{9 + 16\sin 2x} dx$

OR

Evaluate: $\int_{-2}^{4} |x+2|\, dx$

29. Solve the following differential equation:

$(y - \sin^2 x)dx + \tan x\, dy = 0$

OR

Find the general solution of the differential equation:

$(x^3 + y^3)\, dy = x^2 y\, dx$

30. Minimize: $Z = -3x + 4y$

Subject to: $x + 2y \le 8, 3x + 2y \le 12, x \ge 0, y \ge 0$

31. Find: $\int \frac{2x}{(x^2+1)(x^2+2)} dx$

SECTION–D

(This section comprises of long answer type questions (LA) of 5 marks each.)

32. Find the area of the region bounded by the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$.

33. Show that the relation R on $\mathbb{R}$ defined as $R = \{(a, b) : a \leq b\}$, is reflexive and transitive but not symmetric.

34. Find the coordinates of the foot of perpendicular and the length of the perpendicular drawn from the point $P\ (5, 4, 2)$ to the line $\vec{r} = -\hat{i} + 3\hat{j} + \hat{k} + \lambda(2\hat{i} + 3\hat{j} - \hat{k})$. Also find the image of P in this line.

OR

Find the shortest distance between the lines

$$\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1} \text{ and } \frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$$

35. Determine the product $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and use it to solve the system of equations

$x - y + z = 4,\ x - 2y - 2z = 9,\ 2x + y + 3z = 1.$

OR

Solve the following system of equations

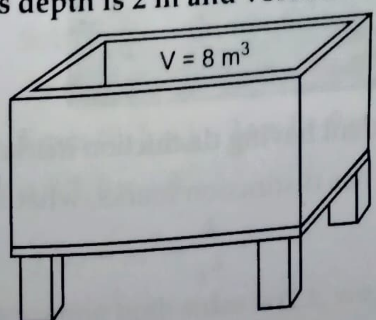
$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4;\quad \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1,\quad \frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$$

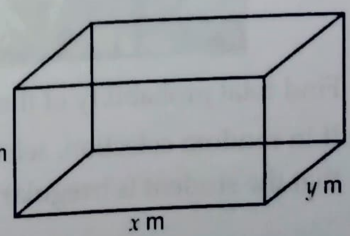
SECTION–E

(This section comprises of 3 case-Study/passage-based questions of 4 marks each. First two case study questions have three sub-parts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two sub-parts of 2 marks each.)

36. **Case Study 1:** Read the following passage and answer the questions given below.

On the request of villagers, a construction agency designs a tank with the help of an architect. Tank consists of rectangular base with rectangular sides, open at the top so that its depth is 2 m and volume is 8 m^3 as shown below:





(*i*) If x and y represent the length and breadth of its rectangular base if the construction of tank cost ₹70 per sq. metre for base and ₹45 per square meter for sides, then express cost function C as a function of x.

(*ii*) Find the critical point of C.

(*iii*) Find the interval in which C is strictly increasing/strictly decreasing.

OR

For minimizing the cost C

(*a*) find the values of x and y.

(*b*) find the minimum cost.

37. Case Study 2: Read the following passage and answer the questions given below.

In the Board Exam for class 12 a function is given $f(x) = x^2 + mx + 3,\ x \in [0, 10]$

(*i*) Is this function continuous in [0, 10]?

(*ii*) If 4 is the critical point of f then find the value of m.

(*iii*) Use First Derivative Test to find x for which f is maximum/minimum.

OR

Use Second Derivative Test to find the point of local minima/local maxima. Also find the points of absolute maxima/absolute minima and absolute maximum/absolute minimum of f.

38. After observing attendance register of class XII, class teacher Shri Mishra comes on conclusion that 30% students have 100% attendance and 70% students are irregular to attend class. When he observed previous year result, he found that 70% of all students who have 100% attendance attain distinction marks while 10% irregular students attain distinction marks. At the end of the year, one student is chosen at random from the class.

(*i*) Find total probability of the selected student having distinction marks.

(*ii*) If in random selection, selected student has distinction marks, what the probability that the student is irregular?