5 SAMPLE PAPER [SOLVED]

Time allowed: 3 hours | Maximum marks: 80

General Instructions: Same as CBSE Sample Question Paper-2024 (Solved).

SECTION–A

Multiple Choice Questions

Each question carries 1 mark.

1. If A is a square matrix, then which of the following matrices is not symmetric?
   (a) $A + A'$ (b) AA' (c) $A'A$ (d) $A - A'$

2. Given that A is a square matrix of order 3 and $|A| = -4$, then $|\text{adj } A|$ is equal to
   (a) -4 (b) 4 (c) -16 (d) 16

3. The vector $\vec{r}$ of magnitude $3\sqrt{2}$ units which makes an angle of $\frac{\pi}{4}$ and $\frac{\pi}{2}$ with y and z-axis, respectively is
   (a) $\vec{r} = \pm 4\hat{i} - 5\hat{j}$ (b) $\vec{r} = \pm 3\hat{i} + 3\hat{j}$ (c) $\vec{r} = \pm 5\hat{i} + 5\hat{j}$ (d) $\vec{r} = \pm 4\hat{i} + 5\hat{j}$

4. If $f(x)$ is everywhere differentiable, then the values of a and b if
$$f(x) = \begin{cases} x^2 + 3x + a, & \text{for } x \le 1 \\ bx + 2, & \text{for } x > 1 \end{cases} \text{ is}$$
   (a) $a = 3, b = 5$ (b) $a = 0, b = 5$ (c) $a = 0, b = 3$ (d) $a = 3, b = 3$

5. If $f'(x) = e^{ax}$, then $f(x)$ is
   (a) ae^{ax} (b) $e^{ax} + C$ (c) e^{ax} (d) $\frac{e^{ax}}{a} + C$

6. The number of solutions of the system of inequations $x + 2y \le 3$, $3x + 4y \ge 12$, $x \ge 0$, $y \ge 1$ is
   (a) 0 (b) 2 (c) finite (d) infinite

7. Area of a rectangle having vertices A, B, C and D with position vectors $-\hat{i} + \frac{1}{2}\hat{j} + 4\hat{k}$, $\hat{i} + \frac{1}{2}\hat{j} + 4\hat{k}$, $\hat{i} - \frac{1}{2}\hat{j} + 4\hat{k}$ and $-\hat{i} - \frac{1}{2}\hat{j} + 4\hat{k}$ respectively is
   (a) 1/2 (b) 1 (c) 2 (d) 4

8. $\int_0^{\frac{2}{3}} \frac{dx}{4 + 9x^2}$ is equal to
   (a) $\frac{\pi}{6}$ (b) $\frac{\pi}{12}$ (c) $\frac{\pi}{24}$ (d) $\frac{\pi}{4}$

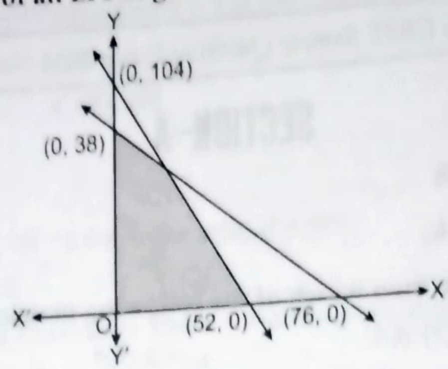
9. **The integrating factor of differential equation $(1-x^2)\frac{dy}{dx} - xy = 1$ is**

(a) $-x$ (b) $\frac{x}{1+x^2}$ (c) $\sqrt{1-x^2}$ (d) $\frac{1}{2}\log(1-x^2)$

10. **If $A = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 59 & 69 & -1 \end{vmatrix}$, then A^{-1}**

(a) is A (b) is $(-A)$ (c) is A^2 (d) does not exist

11. **The feasible region of an LPP is given in the following figure:**



Then, the constraints of the LPP are $x \geq 0, y \geq 0$ and

(a) $2x + y \leq 52$ and $x + 2y \leq 76$ (b) $2x + y \leq 104$ and $x + 2y \leq 76$

(c) $x + 2y \leq 104$ and $2x + y \leq 76$ (d) $x + 2y \leq 104$ and $2x + y \leq 38$

12. **The determinant $\begin{vmatrix} y+k & y & y \\ y & y+k & y \\ y & y & y+k \end{vmatrix}$ is equal to**

(a) $k(3y + k^2)$ (b) $3y + k^3$ (c) $3y + k^2$ (d) $k^2(3y + k)$

13. **If $\begin{vmatrix} 5 & 3 & -1 \\ -7 & x & -3 \\ 9 & 6 & -2 \end{vmatrix} = 0$, then the value of x is**

(a) 3 (b) 5 (c) 7 (d) 9

14. **10% of the bulbs produced in a factory are of red colour and 2% are red and defective. If one bulb is picked up at random, the probability of its being defective, if it is red is**

(a) $\frac{1}{7}$ (b) $\frac{20}{27}$ (c) $\frac{1}{5}$ (d) $\frac{2}{5}$

15. **The general solution of $\frac{dy}{dx} + y\tan x = \sec x$ is**

(a) $y\sec x = \tan x + C$ (b) $y\tan x = \sec x + C$

(c) $\tan x = y\tan x + C$ (d) $x\sec x = \tan y + C$

16. **If $y = 2^x$ then $\frac{d^2y}{dx^2}$ is equal to**

(a) $2^x(\log_e 2)^2$ (b) $2^x\log_e 2$ (c) 2^x (d) None of these

17. If $|\vec{a}| = 8, |\vec{b}| = 3$ and $|\vec{a} \times \vec{b}| = 12$, then value of $\vec{a} \cdot \vec{b}$ is

(a) $\sqrt{3}$ (b) $12\sqrt{3}$ (c) $6\sqrt{3}$ (d) None of these

18. If the line makes an angle of $\frac{\pi}{4}$ with each of y and z axes, then the angle which it makes with x-axis is

(a) 0 (b) π (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

Assertion-Reason Based Questions

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

19. **Assertion** (A) **:** A function $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = \cos x$ is a bijection.

Reason (R) **:** A function $g : A \to B$ is a bijection then $\exists$ a function $h : B \to A$ such that $goh = I_B$ and $hog = I_A$

20. **Assertion** (A) **:** The distance between the points (1, 0, 0) and (0, 0, 0) is 1.

Reason (R) **:** Distance between two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ is

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

SECTION–B

(This section comprises of very short answer type questions (VSA) of 2 marks each.)

21. **Find** the principal values of $\tan^{-1}(-\sqrt{3})$.

OR

Check injectivity and surjectivity of $f : \mathbb{Z} \longrightarrow \mathbb{Z}$ given by $f(x) = x^2$.

22. Find the values of x for which $y = [x(x-2)]^2$ is an increasing function.

23. If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a} + \vec{b}| = |\vec{b}|$, then prove that $(\vec{a} + 2\vec{b})$ is perpendicular to $\vec{a}$.

OR

Show that the line through the points (4, 7, 8), (2, 3, 4) is parallel to line through the points (–1, –2, 1), (1, 2, 5).

24. If $y = (\cos x)^{(\cos x)^{(\cos x)^{\cdots\infty}}}$, then show that $\frac{dy}{dx} = \frac{y^2 \tan x}{y \log \cos x - 1}$.

25. If $\vec{a}$ and $\vec{b}$ are unit vectors and θ is the angle between them, then prove that $\sin\frac{\theta}{2} = \frac{1}{2}|\vec{a} - \vec{b}|$.

SECTION–C

(This section comprises of short answer type questions (SA) of 3 marks each.)

26. Find: $\int \sin^{-1}\sqrt{\frac{x}{a+x}}\,dx$

27. The random variable X can take only the values 0, 1, 2, 3. Given that $P(X = 0) = P(X = 1) = p$ and $P(X = 2) = P(X = 3) = a$ such that $\Sigma p_i x_i^2 = 2\Sigma p_i x_i$, find the value of p.

OR

Four bad oranges are accidentally mixed with 16 good ones. Find the probability distribution of the number of bad oranges when two oranges are drawn at random from this lot. Find the mean of the distribution.

28. Evaluate: $\int_1^3 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{4-x}}\,dx$

OR

Evaluate: $\int_{-1}^{2} |x^3 - 3x^2 + 2x|\,dx$

29. Find the general solution of the following differential equation:

$$x\frac{dy}{dx} = y - x\sin\left(\frac{y}{x}\right)$$

OR

Find the particular solution of the following differential equation, given that $y = 0$ when $x = \frac{\pi}{4}$:

$$\frac{dy}{dx} + y\cot x = \frac{2}{1+\sin x}$$

30. Maximize: $Z = 100x + 120y$

Subject to the constraints: $2x + 3y \leq 30$, $3x + y \leq 17$, $x, y \geq 0$.

31. Find: $\int [\log(\log x) + \frac{1}{(\log x)^2}]\,dx$

SECTION–D

(This section comprises of long answer type questions (LA) of 5 marks each)

32. Using integration, find the area of the region in the first quadrant enclosed by the line $x + y = 2$, the parabola $y^2 = x$ and the x-axis.

OR

Using integration, find the area of the region

$$\{(x, y): 0 \leq y \leq \sqrt{3}x, x^2 + y^2 \leq 4\}$$

33. Find the equation of the line which intersects the lines $\frac{x+2}{1} = \frac{y-3}{2} = \frac{z+1}{4}$ and $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and passes through the point (1, 1, 1).

34. Let $A = \{x \in Z : 0 \leq x \leq 12\}$. Show that

$R = \{(a, b) : a, b \in A, |a - b| \text{ is divisible by } 4\}$ is an equivalence relation. Find the set of all elements related to 1. Also write the equivalence class [2].

35. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix}$, find A^{-1}. Hence, solve the following system of equations:

$x + y + z = 6$, $y + 3z = 11$ and $x - 2y + z = 0$.

OR

If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$, find adj A and verify that $A(\text{adj } A) = (\text{adj } A)A = |A| I_3$.

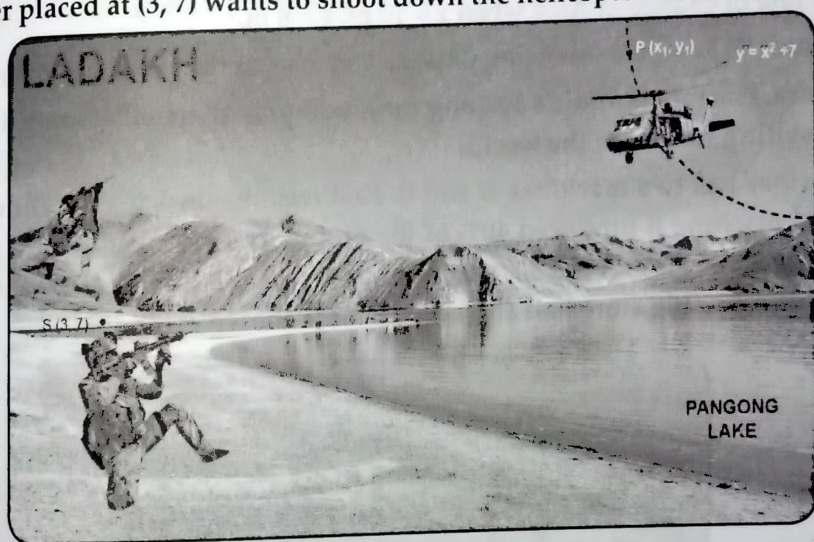
SECTION–E

(This section comprises of 3 case-study/passage-based questions of 4 marks each. First two case study questions have three sub-parts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two sub-parts of 2 marks each.)

36. **Case-Study 1:** Read the following passage and answer the questions given below.

These days chinese and Indian troops are engaged in aggressive melee, face-offs skirmishes at locations near the disputed Pangong Lake in Ladakh.

One day a helicopter of enemy is flying along the curve represented by $y = x^2 + 7$. A soldier placed at (3, 7) wants to shoot down the helicopter when it is nearest to him.



(*i*) If (x_1, y_1) represents the position of the helicopter on the curve $y = x^2 + 7$, when the distance (D) from soldier at S (3, 7) is minimum. Find the relation between x_1, y_1.

(*ii*) Express the distance (D) as a function of x_1 (D is in (*i*)).

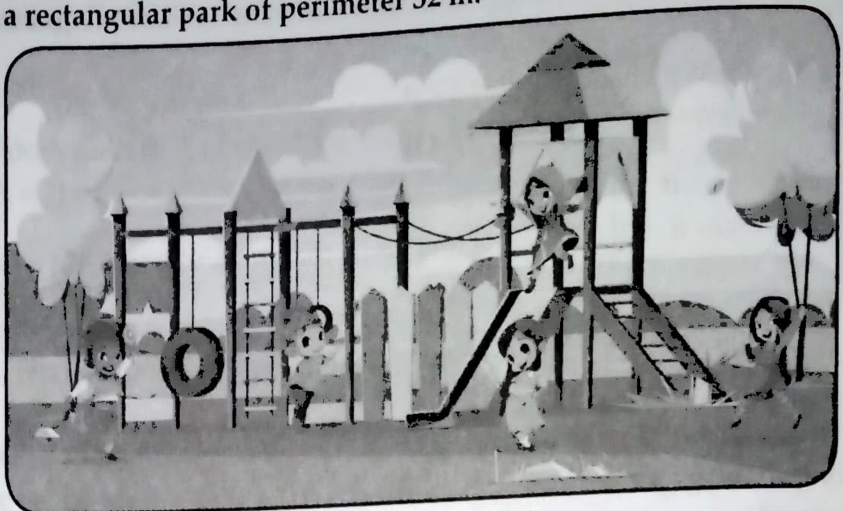
(*iii*) Find the interval in which the function D is strictly increasing/strictly decreasing.

OR

For the function D find the point of local minima and local minimum value of D.

37. **Case-Study 2:** Read the following passage and answer the questions given below.

Given a rectangular park of perimeter 32 m.



(*i*) If length is x cm, breadth is y m then express the area function (A) in terms of x.

(*ii*) Find the critical point for A.

(*iii*) Use first derivative test to find the length and breadth of the park that maximize the area.

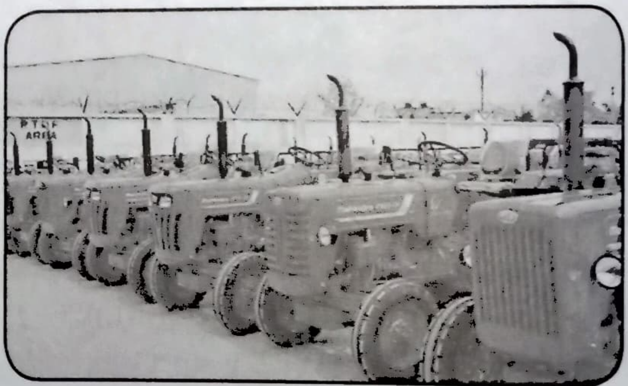
OR

If the rate of change in length x is 3 m/min and that of breadth y in 2 m/min. Find the rate of change in Area when $x = 10$ m, $y = 6$ m.

38. **Case-Study 3:** Read the following passage and answer the questions given below.

Mahindra Tractors is India's leading farm equipment manufacturer. It is the largest tractor selling factory in the world.

This factory has two machines A and B. Past record shows that machine A produced 60% and machine B produced 40% of the output (tractors). Further 2% of the tractors produced by machine A and 1% produced by machine B were defective. All the tractors are put into one big store hall and one tractor is chosen at random.



(*i*) If in random choosing, chosen tractor is defective, what is the probability that the chosen tractor is produced by machine 'A'?

(*ii*) If in random choosing, chosen tractor is defective, what is the probability that the chosen tractor is produced by machine 'B'?